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Quantum projectors and local operators in lattice integrable models

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Abstract
In the framework of the quantum inverse scattering method, we consider a problem of constructing local operators for one-dimensional quantum integrable models, especially for the lattice versions of the nonlinear Schrödinger and sine-Gordon models. We show that a certain class of local operators can be constructed from the matrix elements of the monodromy matrix in a simple way. They are closely related to the quantum projectors and have nice commutation relations with half of the matrix elements of the elementary monodromy matrix. The form factors of these operators can be calculated by using the standard algebraic Bethe ansatz techniques.

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1. Introduction

In one-dimensional quantum integrable models, the quantum inverse scattering method (QISM) [1–5] provides a powerful tool for investigating physical quantities. Among them, the correlation functions have been studied extensively. In order to calculate the correlation functions, it is necessary to deal with states and local operators. At the early stages of the development of QISM, the problem of constructing states was solved by means of the Bethe ansatz.

Recently, great progress was made in constructing local operators in a large class of spin chain models [6–8] which contain the XXX and XXZ spin chains with spin $1/2$. Simple inverse mappings from the matrix elements of the monodromy matrix to the local spin variables were found [7, 8]. The inverse mappings help the calculation of form factors and correlation functions of the spin variables in the framework of QISM [8–10] (see also [11]).

The XXX and XXZ spin chains with spin $1/2$ are fundamental models, i.e. the auxiliary space and the quantum space at a site are isomorphic and the elementary monodromy matrix has a special point at which it becomes the permutation operator for the auxiliary and quantum spaces. The construction of the inverse mapping strongly depends on the existence of the permutation operators. Some non-fundamental models such as higher spin XXX chains were solved by means of the fusion procedure [7]. One characteristic property of these models is that the matrix elements of the elementary monodromy matrix are numerical.

But for some non-fundamental integrable models, such as the lattice nonlinear Schrödinger (LNS) models [12–20] and the lattice sine-Gordon (LSG) models [2, 15, 21–25], the problem of constructing the inverse mapping is not solved. The LNS and LSG models are closely related to the XXX and XXZ spin chains, respectively. Their elementary monodromy matrices are realized by quantum operators and the quantum space at each site is related to an infinite-dimensional representation of the Lie algebra sl_2 or its quantum deformation $U_q(sl_2)$. Only at very specific values of the coupling constant are the infinite-dimensional representations truncated into finite-dimensional ones. The approach by the fusion procedure is possible only at these special points and is very artificial. Moreover, we should take the infinite-dimensional representation limit which has many difficulties.

Therefore, it is better to consider the inverse mapping in a more direct way. This paper is an attempt towards the construction of the inverse mapping.

The form factor bootstrap [26] is one of the approaches for obtaining the correlation functions and was applied to the (continuum limit of) LNS and LSG models. In this approach, the creation operators of the states are Zamolodchikov–Faddeev (ZF) creation operators. The ZF creation-annihilation operators are constructed by using the quantum reflection operators $B(\lambda)A^{-1}(\lambda)$ and their conjugate $D^{-1}(\lambda)C(\lambda)$. The local operators are treated by means of the quantum Gel'fand–Levitan equations [27, 28]. The calculation procedure for the form factors is summarized in the axioms of Smirnov [26]. (See also [29–32] for the approach by the quantum Gel'fand–Levitan equation in case of the quantum nonlinear Schrödinger model).

In contrast to the Gel'fand–Levitan method, we use the reflection operators to construct local operators. In a lattice regularization scheme, the elementary monodromy matrices of LNS and LSG models have special points at which they factorize into quantum projectors [14, 15]. The constructed operators are closely related to these quantum projectors. In this paper, a basis of states is chosen to be the Bethe eigenstates. We show that the form factors of the operators can be calculated by using the algebraic relations in the framework of QISM.

This paper is organized as follows. In section 2, the main idea for constructing the local operators is explained. In section 3, we show that form factors of these local operators can be calculated in the framework of the standard algebraic Bethe ansatz method. Some properties of these local operators are discussed in section 4. The explicit form of the operators is given for the LNS and LSG models in sections 5 and 6, respectively. Section 7 is devoted to discussion.

2. Local operators from quantum projectors

Let $L_n(\lambda)$ $(n = 1, 2, \ldots, N)$ be an infinitesimal monodromy matrix of lattice models with the intertwining property:

$$R(\lambda,\mu)(L_n(\lambda)\otimes L_n(\mu)) = (L_n(\mu)\otimes L_n(\lambda))R(\lambda,\mu). \tag{2.1}$$

Here the numerical R-matrix has the form:

$$R(\lambda,\mu) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b(\lambda,\mu) & c(\lambda,\mu) & 0 \\ 0 & c(\lambda,\mu) & b(\lambda,\mu) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2.2}$$

The functions $b(\lambda, \mu)$ and $c(\lambda, \mu)$ are rational for the LNS model

$$b(\lambda, \mu) = \frac{-\mathrm{i}\kappa}{\lambda - \mu - \mathrm{i}\kappa} \qquad c(\lambda, \mu) = \frac{\lambda - \mu}{\lambda - \mu - \mathrm{i}\kappa} \tag{2.3}$$

and are trigonometric for the LSG model

$$b(\lambda, \mu) = \frac{\mathrm{i}\sin\gamma}{\sinh(\lambda - \mu + \mathrm{i}\gamma)} \qquad c(\lambda, \mu) = \frac{\sinh(\lambda - \mu)}{\sinh(\lambda - \mu + \mathrm{i}\gamma)}. \tag{2.4}$$

The monodromy matrix of the lattice model is given by

$$T(\lambda) = \begin{pmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{pmatrix} = L_N(\lambda) L_{N-1}(\lambda) \cdots L_1(\lambda). \tag{2.5}$$

There are various lattice regularization schemes and several types of $L_n(\lambda)$ with the intertwining property (2.1) have been constructed. Among them, there is a special regularization scheme in which the following factorization property holds: At the points $\lambda = \nu$ where the quantum determinant[1] vanishes, the elementary monodromy matrix factorizes into quantum projectors [14, 15]

$$(L_n(\lambda))_{ij}|_{\lambda=\nu} = P_i(n) Q_j(n) \qquad i, j = 1, 2. \tag{2.6}$$

In this paper, we consider these special models.

Then the monodromy matrix at $\lambda = \nu$ can be written as

$$(T(\nu))_{ij} = P_i(N) W Q_j(1) \tag{2.7}$$

where

$$\begin{aligned} W &= w(N|N-1) w(N-1|N-2) \cdots w(2|1) \\ w(n+1|n) &= \sum_{i=1}^{2} Q_i(n+1) P_i(n). \end{aligned} \tag{2.8}$$

So far, these quantum projectors have been used mainly for constructing the conserved quantities. A simple observation is that these quantum projectors can be used for constructing a certain class of local operators of the lattice models. For example, if $D(\nu)$ is invertible then we have two operators which depend on field variables of site 1 or site N only:

$$D^{-1}(\nu) C(\nu) = (Q_2(1))^{-1} Q_1(1) \qquad B(\nu) D^{-1}(\nu) = P_1(N)(P_2(N))^{-1}. \tag{2.9}$$

For simplicity, we impose the periodic boundary condition: $L_{n+N}(\lambda) = L_n(\lambda)$. Then the shift operator U can be defined by

$$U L_n(\lambda) U^{-1} = L_{n+1}(\lambda) \qquad U|\Omega\rangle = |\Omega\rangle. \tag{2.10}$$

Here $|\Omega\rangle$ is the reference state: $C(\lambda)|\Omega\rangle = 0$. Using this shift operator, we can relate certain local operators to the matrix elements of the monodromy matrix:

$$\begin{aligned} Q_n &:= (Q_2(n))^{-1} Q_1(n) = U^{n-1} D^{-1}(\nu) C(\nu) U^{-n+1} \\ P_n &:= P_1(n)(P_2(n))^{-1} = U^n B(\nu) D^{-1}(\nu) U^{-n}. \end{aligned} \tag{2.11}$$

Off-shell properties of these operators can be extracted from the form factors:

$$\langle\Omega| \left(\prod_{k=1}^{M} C(\mu_k)\right) \mathcal{O}_n \left(\prod_{l=1}^{M'} B(\lambda_l)\right) |\Omega\rangle \qquad \mathcal{O}_n = Q_n \text{ or } P_n. \tag{2.12}$$

Here we assume that the sets of spectral parameters $\{\mu_k\}$ and $\{\lambda_l\}$ satisfy the Bethe equations, respectively.

[1] See equation (4.1) for the definition of the quantum determinant.

We call a state $\prod_k B(\lambda_k)|\Omega\rangle$ the *Bethe state* for generic $\{\lambda_k\}$. When we emphasize that $\{\lambda_k\}$ satisfy the Bethe equations, we call the state the *Bethe eigenstate*.

Let us denote the eigenvalues of the diagonal part of the monodromy matrix on the reference state by

$$(L_n(\lambda))_{11}|\Omega\rangle = a_1(\lambda)|\Omega\rangle \qquad (L_n(\lambda))_{22}|\Omega\rangle = d_1(\lambda)|\Omega\rangle. \tag{2.13}$$

It is known that the Bethe eigenstates are also eigenstates for the shift operator with the following eigenvalues [12, theorem 3]:

$$U\left(\prod_{l=1}^{M} B(\lambda_l)\right)|\Omega\rangle = \left(\prod_{j=1}^{M} r_1(\lambda_j)\right)\left(\prod_{l=1}^{M} B(\lambda_l)\right)|\Omega\rangle \tag{2.14}$$

where $r_1(\lambda) = a_1(\lambda)/d_1(\lambda)$.

Thus, the form factors of Q_n (respectively, P_n) are easily represented by the form factors of $D^{-1}(\nu)C(\nu)$ (respectively, $B(\nu)D^{-1}(\nu)$). These form factors can be calculated by using the algebraic commutation relations.

The time evolution of these operators is controlled by the Hamiltonian operators of the models. The Hamiltonian operator is also diagonalized on the Bethe eigenstates. The form factors of operators at any time can be easily expressed by those of the operators at a certain time (e.g., at $t = 0$). We do not discuss the time evolution in this paper.

Consideration of other points at which $A(\nu)$ is invertible is quite similar. Therefore we omit these cases.

3. Form factors

In this section, we calculate the form factors of $D^{-1}(\nu)C(\nu)$ in a general setting. The calculation for $B(\nu)D^{-1}(\nu)$ is similar. So we omit the case of $B(\nu)D^{-1}(\nu)$.

We forget the lattice structure (2.5) for a while and treat the matrix elements $A(\lambda)$, $B(\lambda)$, $C(\lambda)$ and $D(\lambda)$ as abstract objects. Let

$$A(\lambda)|\Omega\rangle = a(\lambda)|\Omega\rangle \qquad D(\lambda)|\Omega\rangle = d(\lambda)|\Omega\rangle. \tag{3.1}$$

We assume that there is at least one zero for $a(\lambda)$: $a(\nu_A) = 0$. Also, we assume that $D(\nu_A)$ is an invertible operator.

The action of $A(\lambda)$ on the Bethe states is well known:

$$\begin{aligned} A(\mu)\prod_{l=1}^{M} B(\lambda_l)|\Omega\rangle &= a^{(M)}(\mu|\{\lambda_l\})\prod_{l=1}^{M} B(\lambda_l)|\Omega\rangle \\ &\quad + \sum_{j=1}^{M} b^{(M)}(\mu|\lambda_j|\{\lambda_l\}_{l\neq j})B(\mu)\prod_{\substack{l=1\\ l\neq j}}^{M} B(\lambda_l)|\Omega\rangle \end{aligned} \tag{3.2}$$

where

$$\begin{aligned} a^{(M)}(\mu|\{\lambda_l\}) &= a(\mu)\prod_{l=1}^{M} f(\lambda_l,\mu) \\ b^{(M)}(\mu|\lambda_j|\{\lambda_l\}_{l\neq j}) &= a(\lambda_j)g(\mu,\lambda_j)\prod_{\substack{l=1\\ l\neq j}}^{M} f(\lambda_l,\lambda_j). \end{aligned} \tag{3.3}$$

Here $f(\lambda,\mu) = 1/c(\lambda,\mu)$ and $g(\lambda,\mu) = b(\lambda,\mu)/c(\lambda,\mu)$.

For generic μ, λ, we have the following lemma:

$$D^{-1}(\mu)C(\mu)B(\lambda) = f(\lambda,\mu)B(\lambda)D^{-1}(\mu)C(\mu) - g(\lambda,\mu)A(\lambda) + g(\lambda,\mu)D^{-1}(\mu)D(\lambda)(A(\mu) - B(\mu)D^{-1}(\mu)C(\mu)). \tag{3.4}$$

The proof is simple:

$$\begin{aligned} D^{-1}(\mu)C(\mu)B(\lambda) &= D^{-1}(\mu)[C(\mu), B(\lambda)] + D^{-1}(\mu)B(\lambda)D(\mu)D^{-1}(\mu)C(\mu) \\ &= D^{-1}(\mu)g(\lambda,\mu)\left(D(\lambda)A(\mu) - D(\mu)A(\lambda)\right) \\ &\quad + D^{-1}(\mu)\left(f(\lambda,\mu)D(\mu)B(\lambda) - g(\lambda,\mu)D(\lambda)B(\mu)\right)D^{-1}(\mu)C(\mu) \\ &= f(\lambda,\mu)B(\lambda)D^{-1}(\mu)C(\mu) - g(\lambda,\mu)A(\lambda) \\ &\quad + g(\lambda,\mu)D^{-1}(\mu)D(\lambda)(A(\mu) - B(\mu)D^{-1}(\mu)C(\mu)). \end{aligned} \tag{3.5}$$

Then using this lemma and by induction, we can prove that $D^{-1}(\nu_A)C(\nu_A)$ acts on the right Bethe states as follows:

$$D^{-1}(\nu_A)C(\nu_A)\prod_{l=1}^{M} B(\lambda_l)|\Omega\rangle = \sum_{j=1}^{M} b^{(M)}(\nu_A|\lambda_j|\{\lambda_l\}_{l\neq j}) \prod_{\substack{l=1\\ l\neq j}}^{M} B(\lambda_l)|\Omega\rangle. \tag{3.6}$$

This is quite similar to the action of the nonlinear Schrödinger field $\Psi(0)$ on the Bethe states [33]. Therefore, the calculation procedure for the form factors of $\Psi(0)$ [34, 35] can also be applied to the following form factors:

$$F_M := \langle\Omega|\left(\prod_{k=1}^{M} C(\mu_k)\right) D^{-1}(\nu_A)C(\nu_A)\left(\prod_{l=1}^{M+1} B(\lambda_l)\right)|\Omega\rangle \Big/ \langle\Omega|\Omega\rangle. \tag{3.7}$$

Here $\{\mu_k\}$ and $\{\lambda_l\}$ are solutions of the Bethe equations, respectively.

After some calculations which are a slight modification of [34, 35], we have

$$\begin{aligned} F_M = {} & \prod_{k=1}^{M+1}\prod_{l=1}^{M+1} h(\lambda_k,\lambda_l) \prod_{1\leqslant k<l\leqslant M} g(\mu_l,\mu_k) \prod_{1\leqslant k<l\leqslant M+1} g(\lambda_k,\lambda_l) \\ & \times \prod_{l=1}^{M} d(\mu_l) \prod_{l=1}^{M+1} d(\lambda_l) \left(\sum_{j=1}^{M+1}(-1)^{j-1} g(\nu_A,\lambda_j)\det{}_M S^{(j)}\right). \end{aligned} \tag{3.8}$$

Here $S^{(j)}$ $(j = 1, 2, \ldots, M+1)$ is an $M \times M$ matrix obtained by removing the jth row from an $(M+1)\times M$ matrix S whose matrix elements are defined by

$$S_{kl} = t(\lambda_k,\mu_l)\frac{\prod_{m=1}^{M} h(\lambda_k,\mu_m)}{\prod_{m=1}^{M+1} h(\lambda_k,\lambda_m)} - t(\mu_l,\lambda_k)\frac{\prod_{m=1}^{M} h(\mu_m,\lambda_k)}{\prod_{m=1}^{M+1} h(\lambda_m,\lambda_k)} \tag{3.9}$$

$(k = 1, 2, \ldots, M+1, l = 1, 2, \ldots, M)$. Also, $t(\lambda,\mu) = b^2(\lambda,\mu)/c(\lambda,\mu)$ and $h(\lambda,\mu) = 1/b(\lambda,\mu)$.

In the following, we will show that the sum on the right-hand side of equation (3.8) can be rewritten by using a single determinant.

By using the Cauchy determinant identity or by evaluating the residues, we can prove the following identity:

$$\sum_{j=1}^{M+1} g(\eta,\lambda_j)\xi_j = \prod_{j=1}^{M+1} g(\eta,\lambda_j) \prod_{i=1}^{M} \frac{1}{g(\eta,\mu_i)} \tag{3.10}$$

where

$$\xi_k := \prod_{\substack{l=1\\ l\neq k}}^{M+1} g(\lambda_l, \lambda_k) \prod_{l=1}^{M} \frac{1}{g(\mu_l, \lambda_k)}. \tag{3.11}$$

With the help of this identity, it is possible to check that the $(M + 1)$-dimensional vector ξ_k is a left null vector of the matrix S: $\sum_{k=1}^{M+1} \xi_k S_{kl} = 0$. The substitution of $S_{M+1,l} = -\sum_{k=1}^{M} (\xi_k/\xi_{M+1}) S_{kl}$ into $\det_M S^{(j)}$ leads to

$$(-1)^{j-1} \det_M S^{(j)} = (-1)^M \frac{\xi_j}{\xi_{M+1}} \det_M S^{(M+1)}. \tag{3.12}$$

In other words, the combination of $(-1)^{j-1} \det_M S^{(j)}/\xi_j$ is a j-independent quantity:

$$\begin{aligned}(\det_M S^{(1)})/\xi_1 &= -(\det_M S^{(2)})/\xi_2 = \cdots \\ &= (-1)^{j-1}(\det_M S^{(j)})/\xi_j = \cdots = (-1)^M (\det_M S^{(M+1)})/\xi_{M+1}.\end{aligned} \tag{3.13}$$

By virtue of equations (3.12) and (3.10) for $\eta = \nu_A$, we have the final result for the form factors:

$$\begin{aligned}&\langle\Omega| \left(\prod_{l=1}^{M} C(\mu_l)\right) D^{-1}(\nu_A) C(\nu_A) \left(\prod_{l=1}^{M+1} B(\lambda_l)\right) |\Omega\rangle \Big/ \langle\Omega|\Omega\rangle \\ &\quad = (-1)^M \det_M (S_{kl})_{1\leqslant k,l\leqslant M} \prod_{l=1}^{M} \mathrm{d}(\mu_l) \prod_{l=1}^{M+1} \mathrm{d}(\lambda_l) \prod_{k=1}^{M+1}\prod_{l=1}^{M+1} h(\lambda_k, \lambda_l) \\ &\quad\quad \times \prod_{1\leqslant k<l\leqslant M} g(\mu_l, \mu_k) \prod_{1\leqslant k<l\leqslant M} g(\lambda_k, \lambda_l) \prod_{l=1}^{M} g(\mu_l, \lambda_{M+1}) \frac{\prod_{j=1}^{M+1} g(\nu_A, \lambda_j)}{\prod_{i=1}^{M} g(\nu_A, \mu_i)}.\end{aligned} \tag{3.14}$$

Here we have used the relation $\det_M S^{(M+1)} = \det_M (S_{kl})_{1\leqslant k,l\leqslant M}$.

4. Some properties of Q_n and P_n

In this section, we discuss some properties of the local operators Q_n and P_n.

For a spectral parameter μ, let us define $\mu^\vee := \mu + \mathrm{i}\kappa$ for the rational case and $\mu^\vee := \mu - \mathrm{i}\gamma$ for the trigonometric case. Then

$$T(\mu)\sigma_2 T^t(\mu^\vee)\sigma_2 = \det_q(T(\mu)) I_2. \tag{4.1}$$

Here t denotes the transpose for the auxiliary space and $\det_q(T(\mu))$ is a central element called the *quantum determinant*.

Lemma (3.4) can be rewritten as follows:

$$\begin{aligned}D^{-1}(\mu)C(\mu)B(\lambda) &= f(\lambda, \mu)B(\lambda)D^{-1}(\mu)C(\mu) - g(\lambda, \mu)A(\lambda) \\ &\quad + g(\lambda, \mu)D(\lambda)D^{-1}(\mu)D^{-1}(\mu^\vee)\det_q(T(\mu)).\end{aligned} \tag{4.2}$$

At $\mu = \nu_A$, the quantum determinant vanishes in the module constructed over the reference state. Without loss of generality, we can set $\det_q(T(\nu_A)) = 0$.

The following relation comes from the intertwining property:

$$D^{-1}(\mu)C(\mu)D(\lambda) = f(\lambda, \mu)D(\lambda)D^{-1}(\mu)C(\mu) - g(\lambda, \mu)C(\lambda). \tag{4.3}$$

Now let us recall the lattice structure (2.5) and the definition of the local operator $Q_1 = D^{-1}(\nu_A)C(\nu_A)$. From equations (4.2) and (4.3), we immediately have

$$Q_1 B(\lambda) = f(\lambda, \nu_A)B(\lambda)Q_1 - g(\lambda, \nu_A)A(\lambda) \tag{4.4}$$

$$Q_1 D(\lambda) = f(\lambda, \nu_A) D(\lambda) Q_1 - g(\lambda, \nu_A) C(\lambda). \tag{4.5}$$

It turns out that these relations arise as a consequence of

$$Q_1 (L_1(\lambda))_{i2} = f(\lambda, \nu_A)(L_1(\lambda))_{i2} Q_1 - g(\lambda, \nu_A)(L_1(\lambda))_{i1} \qquad i = 1, 2. \tag{4.6}$$

Applying the shift operator to this equation, we have

$$Q_n (L_n(\lambda))_{i2} = f(\lambda, \nu_A)(L_n(\lambda))_{i2} Q_n - g(\lambda, \nu_A)(L_n(\lambda))_{i1}. \tag{4.7}$$

Compared to the action of $Q_1 = D^{-1}(\nu_A)C(\nu_A)$ on the right Bethe states (3.6), the action on the left Bethe states is complicated. For a spectral parameter μ, let $\mu^{(m)} := \mu + \mathrm{i}m\kappa$ for the LNS model and $\mu^{(m)} := \mu - \mathrm{i}m\gamma$ for the LSG model. (Note that $\mu^{(1)} = \mu^{\vee}$.) From

$$\begin{aligned}\langle\Omega| \prod_{k=1}^{M} C(\mu_k) D(\lambda) &= d^{(M)}(\lambda|\{\mu_k\})\langle\Omega| \prod_{k=1}^{M} C(\mu_k) \\ &\quad + \sum_{j=1}^{M} c^{(M)}(\lambda|\mu_j|\{\mu_k\}_{k\neq j})\langle\Omega| C(\lambda) \prod_{\substack{k=1\\k\neq j}}^{M} C(\mu_k)\end{aligned} \tag{4.8}$$

$$d^{(M)}(\lambda|\{\mu_k\}) = d(\lambda)\prod_{k=1}^{M} f(\lambda, \mu_k) \qquad c^{(M)}(\lambda|\mu_j|\{\mu_k\}_{k\neq j}) = d(\mu_j) g(\mu_j, \lambda) \prod_{\substack{k=1\\k\neq j}}^{M} f(\mu_j, \mu_k) \tag{4.9}$$

we have

$$\begin{aligned}\langle\Omega| \left(\prod_{k=1}^{M} C(\mu_k)\right) D^{-1}(\lambda) &= (d^{(M)}(\lambda|\{\mu_k\}))^{-1}\langle\Omega| \left(\prod_{k=1}^{M} C(\mu_k)\right) \\ &\quad - \sum_{j=1}^{M} \frac{c^{(M)}(\lambda|\mu_j|\{\mu_k\}_{k\neq j})}{d^{(M)}(\lambda|\{\mu_k\})}\langle\Omega| \left(\prod_{\substack{k=1\\k\neq j}}^{M} C(\mu_k)\right) D^{-1}(\lambda^{(-1)}) C(\lambda^{(-1)}).\end{aligned} \tag{4.10}$$

Here we have used $C(\lambda)D^{-1}(\lambda) = D^{-1}(\lambda^{(-1)})C(\lambda^{(-1)})$. If we use these relations recursively, we can see that the result of the action of $D^{-1}(\nu_A)C(\nu_A)$ on the left Bethe state yields terms which contain operators $C\big(\nu_A^{(-m)}\big)$ for $m = 0, 1, \ldots, M$.

The origin of this complicated action is the following commutation relation:

$$C(\mu)D^{-1}(\lambda) = c(\lambda, \mu)D^{-1}(\lambda)C(\mu) + b(\lambda, \mu)D^{-1}(\lambda)D(\mu)D^{-1}(\lambda^{(-1)})C(\lambda^{(-1)}) \tag{4.11}$$

which can be derived from the intertwining properties.

To conclude, the local operator Q_n has nice commutation relations with half of the matrix elements of the infinitesimal monodromy matrix.

Similarly, from

$$\begin{aligned}C(\lambda)B(\mu)D^{-1}(\mu) &= f(\lambda, \mu)B(\mu)D^{-1}(\mu)C(\lambda) - g(\lambda, \mu)A(\lambda) \\ &\quad + g(\lambda, \mu)\,\mathrm{det}_q(T(\mu))D^{-1}(\mu^{\vee})D^{-1}(\mu)D(\lambda)\end{aligned} \tag{4.12}$$

$$D(\lambda)B(\mu)D^{-1}(\mu) = f(\lambda, \mu)B(\mu)D^{-1}(\mu)D(\lambda) - g(\lambda, \mu)B(\lambda) \tag{4.13}$$

we can derive the following property of the local operator $P_n = U^n B(\nu_A)D^{-1}(\nu_A)U^{-n}$:

$$(L_n(\lambda))_{2j} P_n = f(\lambda, \nu_A) P_n (L_n(\lambda))_{2j} - g(\lambda, \nu_A)(L_n(\lambda))_{1j} \qquad j = 1, 2. \tag{4.14}$$

The operator P_N acts nicely on the left Bethe states and has a complicated action on the right Bethe states.

5. Lattice nonlinear Schrödinger model

The Hamiltonian of the quantum nonlinear Schrödinger model is given by

$$H^{(\mathrm{NLS})} = \int \mathrm{d}x \left(\frac{\partial \psi^*}{\partial x} \frac{\partial \psi}{\partial x} + \kappa \psi^* \psi^* \psi \psi \right). \tag{5.1}$$

Various types of LNS models have been proposed [12–20].

For simplicity, we use the LNS model of [14, 15] as an example. Let us put the system in a box of length $2L$: $(-L < x \leqslant L)$, and discretize it to the lattice with N-sites: $x_n = -L + n\Delta$ $(n = 1, 2, \ldots, N)$. Here the lattice spacing is given by $\Delta = 2L/N$. The elementary operators for this lattice model are constructed from original fields as follows:

$$\psi_n = \int_{x_n - \Delta}^{x_n} \psi(x, 0)\,\mathrm{d}x \qquad \psi_n^* = \int_{x_n - \Delta}^{x_n} \psi^*(x, 0)\,\mathrm{d}x. \tag{5.2}$$

They satisfy the canonical commutation relations: $[\psi_m, \psi_n^*] = \Delta \delta_{m,n}$.

The infinitesimal monodromy matrix is given by [14, 15]

$$L_n(\lambda) = \begin{pmatrix} 1 - \mathrm{i}(\lambda/2)\Delta + (\kappa/2)\psi_n^* \psi_n & \sqrt{\kappa}\psi_n^*(1 + (\kappa/4)\psi_n^*\psi_n)^{1/2} \\ \sqrt{\kappa}(1 + (\kappa/4)\psi_n^*\psi_n)^{1/2}\psi_n & 1 + \mathrm{i}(\lambda/2)\Delta + (\kappa/2)\psi_n^*\psi_n \end{pmatrix}. \tag{5.3}$$

For example, at $\lambda = \nu_A := -2\mathrm{i}/\Delta$, the infinitesimal monodromy matrix factorizes into the quantum projectors [14, 15]: $(L_n(\nu_A))_{ij} = P_i(n) Q_j(n)$ where

$$P_1(n) = \sqrt{\kappa/2}\psi_n^* \qquad P_2(n) = \sqrt{2}(1 + (\kappa/4)\psi_n^*\psi_n)^{1/2} \tag{5.4}$$

$$Q_1(n) = \sqrt{\kappa/2}\psi_n \qquad Q_2(n) = \sqrt{2}(1 + (\kappa/4)\psi_n^*\psi_n)^{1/2}. \tag{5.5}$$

Thus, for generic coupling constant κ,

$$w(n+1|n) = 2(1 + (\kappa/4)\psi_{n+1}^*\psi_{n+1})^{1/2}(1 + (\kappa/4)\psi_n^*\psi_n)^{1/2} + (\kappa/2)\psi_{n+1}\psi_n^* \tag{5.6}$$

is an invertible operator. There exists $D^{-1}(\nu_A)$.

The corresponding local operators are

$$Q_n = \frac{\sqrt{\kappa}}{2}(1 + (\kappa/4)\psi_n^*\psi_n)^{-1/2}\psi_n \qquad P_n = \frac{\sqrt{\kappa}}{2}\psi_n^*(1 + (\kappa/4)\psi_n^*\psi_n)^{-1/2}. \tag{5.7}$$

In the continuum limit $\Delta \to 0$, they become the field operators of the quantum nonlinear Schrödinger model:

$$Q_n \to \frac{\sqrt{\kappa}}{2}\Delta\psi(x, 0) \qquad P_n \to \frac{\sqrt{\kappa}}{2}\Delta\psi^*(x, 0) \qquad x = -L + n\Delta \tag{5.8}$$

and the form factors (3.14) give consistent results with [35].

6. Application to the lattice sine-Gordon model

The Hamiltonian of the quantum sine-Gordon model is given by

$$H^{(\mathrm{SG})} = \int \mathrm{d}x \left(\frac{1}{2}\pi^2 + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{m^2}{\beta^2}(1 - \cos\beta u) \right). \tag{6.1}$$

There are several approaches to the lattice regularization of the sine-Gordon model [15, 21–25]. For simplicity, we use the regularization scheme proposed in [15, 21, 22].

The infinitesimal monodromy matrix is given by [15, 21, 22]

$$L_n(\lambda) = \begin{pmatrix} \pi_n^{-1/2}\varphi(u_n)\pi_n^{-1/2} & -\mathrm{i}(m\Delta/2)\sin((\beta/2)u_n + \mathrm{i}\lambda) \\ -\mathrm{i}(m\Delta/2)\sin((\beta/2)u_n - \mathrm{i}\lambda) & \pi_n^{1/2}\varphi(u_n)\pi_n^{1/2} \end{pmatrix} \tag{6.2}$$

$$\pi_n = \exp\left(\frac{\mathrm{i}}{4}\beta p_n\right) \qquad \varphi(u_n) = (1 + 2r\cos\beta u_n)^{1/2} \qquad r = \left(\frac{m\Delta}{4}\right)^2. \tag{6.3}$$

Here $\gamma = \beta^2/8$ and the lattice operators u_n and p_n ($n = 1, 2, \ldots, N$) satisfy the canonical commutation relations: $[u_n, p_m] = \mathrm{i}\delta_{nm}$. In the continuum limit $\Delta \to 0$, $u_n \to u(x)$, $p_n \to \pi(x)\Delta$ $(-L < x = -L + n\Delta \leqslant L)$.

In order to construct the reference state $|\Omega\rangle$, the elementary monodromy matrix should be taken as a composite of the infinitesimal monodromy matrices of two adjacent sites [2]:

$$\mathcal{L}_k(\lambda) = L_{2k}(\lambda)L_{2k-1}(\lambda) \qquad k = 1, 2, \ldots, N/2. \tag{6.4}$$

Here we assume the number of sites N is even. Then

$$(\mathcal{L}_k(\lambda))_{11}|\Omega\rangle = a_1(\lambda)|\Omega\rangle \qquad (\mathcal{L}_k(\lambda))_{22}|\Omega\rangle = d_1(\lambda)|\Omega\rangle \tag{6.5}$$

$$a_1(\lambda) = 1 + 2r\cosh(2\lambda - \mathrm{i}\gamma) \qquad d_1(\lambda) = 1 + 2r\cosh(2\lambda + \mathrm{i}\gamma). \tag{6.6}$$

Thus, the shift operator is defined by

$$U\mathcal{L}_kU^{-1} = \mathcal{L}_{k+1} \qquad U|\Omega\rangle = |\Omega\rangle. \tag{6.7}$$

In other words, it acts as a two-site shift for the local variables: $Uu_nU^{-1} = u_{n+2}$, $Up_nU^{-1} = p_{n+2}$. In general, the periodic boundary condition has the form: $u_{n+N} = u_n + (2\pi/\beta)\mathcal{Q}$, $p_{n+N} = p_n$ where $\mathcal{Q}$ is the topological charge. In this paper, we only consider the sector with $\mathcal{Q} = 0$ for simplicity.

Let us introduce a positive 'momentum cutoff' parameter Λ by $2r\cosh\Lambda = 1$.

At $\lambda = \nu_A^{(\epsilon,\epsilon')} := \frac{1}{2}(\mathrm{i}\gamma + \epsilon\Lambda + \mathrm{i}\epsilon'\pi)$ $(\epsilon, \epsilon' = \pm 1)$, $a_1(\lambda)$ vanishes and the infinitesimal monodromy matrix factorizes into the quantum projectors [14, 15]:

$$\left(L_n\left(\nu_A^{(\epsilon,\epsilon')}\right)\right)_{ij} = P_i^{(\epsilon,\epsilon')}(n)Q_j^{(\epsilon,\epsilon')}(n) \tag{6.8}$$

$$\begin{aligned}
P_1^{(\epsilon,\epsilon')}(n) &= (4r)^{1/4}\pi_n^{-1/2}\left[\cos\tfrac{1}{2}(\beta u_n + i\epsilon\Lambda)\right]^{1/2}\\
P_2^{(\epsilon,\epsilon')}(n) &= -i\epsilon'(4r)^{1/4}\pi_n^{1/2}\left[\cos\tfrac{1}{2}(\beta u_n - i\epsilon\Lambda)\right]^{1/2}\\
Q_1^{(\epsilon,\epsilon')}(n) &= (4r)^{1/4}\left[\cos\tfrac{1}{2}(\beta u_n - i\epsilon\Lambda)\right]^{1/2}\pi_n^{-1/2}\\
Q_2^{(\epsilon,\epsilon')}(n) &= i\epsilon'(4r)^{1/4}\left[\cos\tfrac{1}{2}(\beta u_n + i\epsilon\Lambda)\right]^{1/2}\pi_n^{1/2}.
\end{aligned}$$

These quantum projectors are invertible operators. From $(\mathcal{L}_k(\lambda))_{21}|\Omega\rangle = 0$, we can see that the operator

$$w^{(\epsilon,\epsilon')}(2k|2k-1) = \sum_{i=1}^{2} Q_i^{(\epsilon,\epsilon')}(2k)P_i^{(\epsilon,\epsilon')}(2k-1) \tag{6.9}$$

has a zero eigenvalue. So, $w^{(\epsilon,\epsilon')}(2k|2k-1)$ is not invertible and consequently, $D\left(\nu_A^{(\epsilon,\epsilon')}\right)$ is also not invertible. The assumption in section 3 that $D(\nu_A)$ has an inverse does not hold. We should modify the argument of section 3.

Although $D^{-1}\left(\nu_A^{(\epsilon,\epsilon')}\right)$ does not exist, we can define the following unitary operators:

$$P_n^{(\epsilon)} := \epsilon' P_1^{(\epsilon,\epsilon')}(n)\left(P_2^{(\epsilon,\epsilon')}(n)\right)^{-1} \qquad Q_n^{(\epsilon)} := \epsilon'\left(Q_2^{(\epsilon,\epsilon')}(n)\right)^{-1}Q_1^{(\epsilon,\epsilon')}(n). \tag{6.10}$$

The left-hand sides of the above equations do not depend on a choice of $\epsilon' = \pm 1$. Without loss of generality, we set $\epsilon' = 1$. Let $\nu_A^{(\epsilon)} := \nu_A^{(\epsilon,+1)}$.

Let us introduce the following unitary operators:

$$\mathcal{O}_n^{(\epsilon)} := \pi_n^{-1/2}\left[\frac{\cos(1/2)(\beta u_n - \mathrm{i}\epsilon\Lambda)}{\cos(1/2)(\beta u_n + \mathrm{i}\epsilon\Lambda)}\right]^{1/2}\pi_n^{-1/2} \qquad \epsilon = \pm 1. \tag{6.11}$$

Then $P_n^{(\epsilon)} = \mathrm{i}\mathcal{O}_n^{(-\epsilon)}$ and $Q_n^{(\epsilon)} = -\mathrm{i}\mathcal{O}_n^{(\epsilon)}$.

The unitary operators (6.11) have a complicated form. But in the continuum limit, they are simplified into exponential operators[2]:

$$\lim_{\Delta\to 0}\mathcal{O}_n^{(\epsilon)} = \mathrm{e}^{(1/2)\,\mathrm{i}\epsilon(\beta u(x)-\gamma)} \qquad \epsilon = \pm 1. \tag{6.12}$$

Thus, $O_n^{(\epsilon)}$ is interpreted as a lattice regularization of the exponential operator $\mathrm{e}^{(1/2)\,\mathrm{i}\epsilon\beta u(x)}$.

By using the explicit expressions, we can check that these lattice 'exponential operators' satisfy equations (4.7) and (4.14).

Therefore, in place of $D^{-1}(\nu_A)C(\nu_A)$, we can use the well-defined operator $Q_1^{(\epsilon)}$. Because the unitary operator $Q_1^{(\epsilon)}$ does not annihilate the reference state, the action of $Q_1^{(\epsilon)}$ on the Bethe state has an 'anomalous' term:

$$\begin{aligned} Q_1^{(\epsilon)}\prod_{l=1}^{M+1}B(\lambda_l)|\Omega\rangle &= \sum_{j=1}^{M+1} b^{(M+1)}\left(\nu_A^{(\epsilon)}|\lambda_j|\{\lambda_l\}_{l\neq j}\right)\prod_{\substack{l=1\\ l\neq j}}^{M+1}B(\lambda_l)|\Omega\rangle \\ &\quad + \left(\prod_{k=1}^{M+1} f\left(\lambda_k,\nu_A^{(\epsilon)}\right)\right)\left(\prod_{l=1}^{M+1}B(\lambda_l)\right)Q_1^{(\epsilon)}|\Omega\rangle. \end{aligned} \tag{6.13}$$

But because

$$\langle\Omega|\left(\prod_{k=1}^{M}C(\mu_k)\right)\left(\prod_{l=1}^{M+1}B(\lambda_l)\right) = 0 \tag{6.14}$$

the anomalous term gives no contribution to the form factors

$$\langle\Omega|\left(\prod_{k=1}^{M}C(\mu_k)\right)Q_1^{(\epsilon)}\left(\prod_{l=1}^{M+1}B(\lambda_l)\right)|\Omega\rangle. \tag{6.15}$$

Formula (3.14) gives the correct result even for $Q_1^{(\epsilon)}$.

It may seem that the anomalous term would contribute to the form factors for the same numbers of $C(\mu)$ and $B(\lambda)$. If we recall

$$\langle\Omega|\left(\prod_{k=1}^{M}C(\mu_k)\right)\left(\prod_{l=1}^{M}B(\lambda_l)\right) = \mathcal{N}^{(M)}(\{\lambda_l\})\delta_{\{\mu_k\},\{\lambda_l\}}\langle\Omega| \tag{6.16}$$

where $\mathcal{N}^{(M)}(\{\lambda_l\})$ is the norm of the Bethe eigenstate:

$$\mathcal{N}^{(M)}(\{\lambda_l\}) = \langle\Omega|\left(\prod_{k=1}^{M}C(\lambda_k)\right)\left(\prod_{l=1}^{M}B(\lambda_l)\right)|\Omega\rangle \Big/ \langle\Omega|\Omega\rangle \tag{6.17}$$

(for the explicit form, see [36, 37]), then

$$\begin{aligned} &\langle\Omega|\left(\prod_{k=1}^{M}C(\mu_k)\right)Q_1^{(\epsilon)}\left(\prod_{l=1}^{M}B(\lambda_l)\right)|\Omega\rangle \\ &\quad = \delta_{\{\mu_k\},\{\lambda_l\}}\mathcal{N}^{(M)}(\{\lambda_l\})\left(\prod_{l=1}^{M} f\left(\lambda_l,\nu_A^{(\epsilon)}\right)\right)\langle\Omega|Q_1^{(\epsilon)}|\Omega\rangle = 0. \end{aligned} \tag{6.18}$$

$\langle\Omega|Q_1^{(\epsilon)}|\Omega\rangle$ vanishes due to the factor $\exp(-\mathrm{i}(\beta/4)p_1)$ in $Q_1^{(\epsilon)}$.

[2] There is a subtlety due to the operator ordering. One must choose a prescription for the limit. Here we take the continuum limit after 'normal-ordering' the operator (6.11) (i.e. moving p_n to the right of all u_n).

We conclude that although $D^{-1}\big(\nu_A^{(\epsilon)}\big)$ does not exist, the result of section 3 is still correct for the LSG model. Thus, as a mnemonic, we can write:

$$Q_1^{(\epsilon)} = D^{-1}\big(\nu_A^{(\epsilon)}\big)C\big(\nu_A^{(\epsilon)}\big). \tag{6.19}$$

By means of this mnemonic, we can make it clear that the operators (6.11) have different character depending on whether n is even or not:

$$\mathcal{O}_{2k}^{(\epsilon)} = -\mathrm{i}U^k B\big(\nu_A^{(-\epsilon)}\big)D^{-1}\big(\nu_A^{(-\epsilon)}\big)U^{-k} \qquad \mathcal{O}_{2k+1}^{(\epsilon)} = \mathrm{i}U^k D^{-1}\big(\nu_A^{(\epsilon)}\big)C\big(\nu_A^{(\epsilon)}\big)U^{-k}. \tag{6.20}$$

The operators at even sites (respectively, odd sites) are creation-type (respectively, annihilation-type) operators.

7. Discussion

In this paper, we showed that a certain class of local operators can be constructed by using the quantum projectors. For the LNS model, these local operators (5.7) are lattice analogues of the continuum nonlinear Schrödinger fields $\psi(x)$ and $\psi^*(x)$. For the LSG model, these local operators (6.11) are lattice analogues of the exponential operators $\mathrm{e}^{(1/2)\,\mathrm{i}\epsilon\beta u(x)}$.

These local operators have a complicated form. But this lattice regularization simplifies the calculation of the form factors. The form factors of these lattice operators can be calculated only by using well-established techniques of the algebraic Bethe ansatz.

In addition, because the inputs are the elementary monodromy matrix $L_n(\lambda)$ (5.3) or (6.2) which are already complicated, it seems unavoidable to have complicated output if one tries to keep simplicity of the inverse mapping.

For the LSG model, we considered the local operators $\mathcal{O}_n^{(\epsilon)}$ in the sector of the zero topological charge $\mathcal{Q}$. Consideration of the sector $\mathcal{Q} \neq 0$ is necessary. Moreover, in order to make connection with the quantum sine-Gordon model, we should consider the thermodynamic limit. Note that the following continuum limits have no ambiguity of the operator ordering:

$$\lim_{\Delta\to 0} \mathrm{e}^{\mathrm{i}\epsilon\gamma}\mathcal{O}_n^{(\epsilon)}\big(\mathcal{O}_n^{(-\epsilon)}\big)^{-1} = \lim_{\Delta\to 0} \mathrm{e}^{-\mathrm{i}\epsilon\gamma}\big(\mathcal{O}_n^{(-\epsilon)}\big)^{-1}\mathcal{O}_n^{(\epsilon)} = \mathrm{e}^{\mathrm{i}\epsilon\beta u(x)}. \tag{7.1}$$

Therefore, in principle, the form factors of the exponential operators $\mathrm{e}^{\pm\mathrm{i}\beta u(x)}$ can be evaluated using those of $\mathcal{O}_n^{(\epsilon)}$. Also, the results may be used to consider the form factors in the finite volume.

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